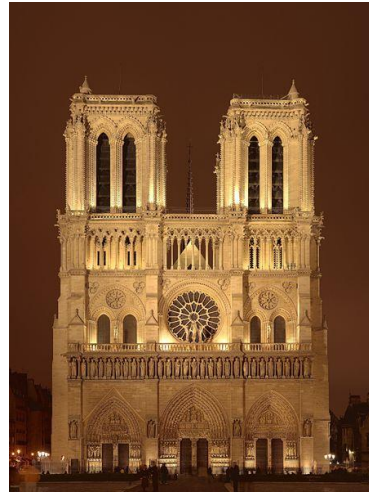
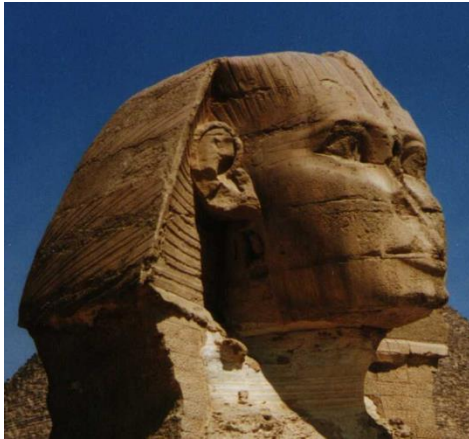


Lambertian model of reflectance I: shape from shading and photometric stereo

Ronen Basri

Weizmann Institute of Science

Variations due to lighting (and pose)



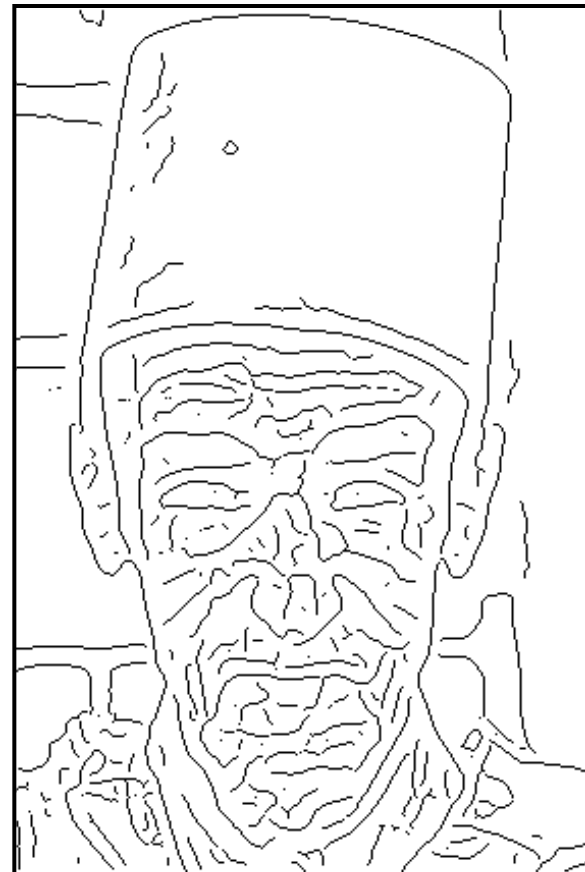


Dumitru Verdianu
Flying Pregnant Woman

Edges



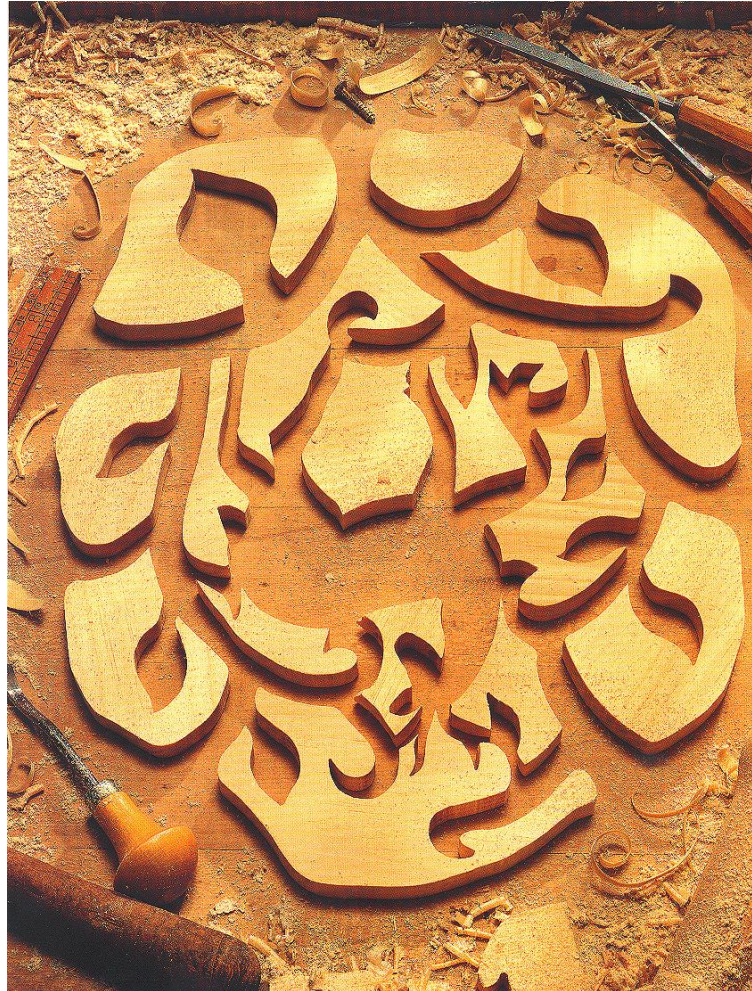
Edges on smooth surfaces



Bump or dip?



What sticks out?

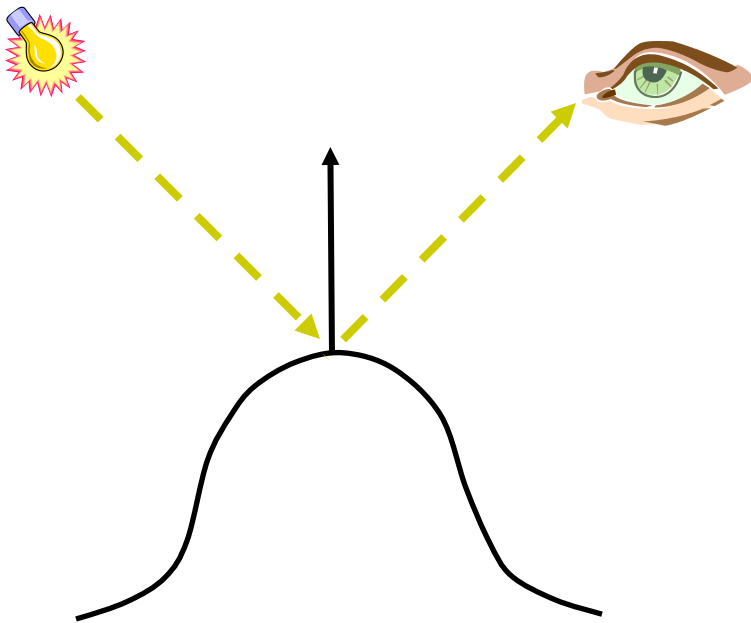


Light field

- Rays travel from sources to objects
- There they are either absorbed or reflected
- Energy of light decreases with distance and number of bounces
- Camera is designed to capture the set of rays that travel through the focal center
- We will assume the camera response is linear

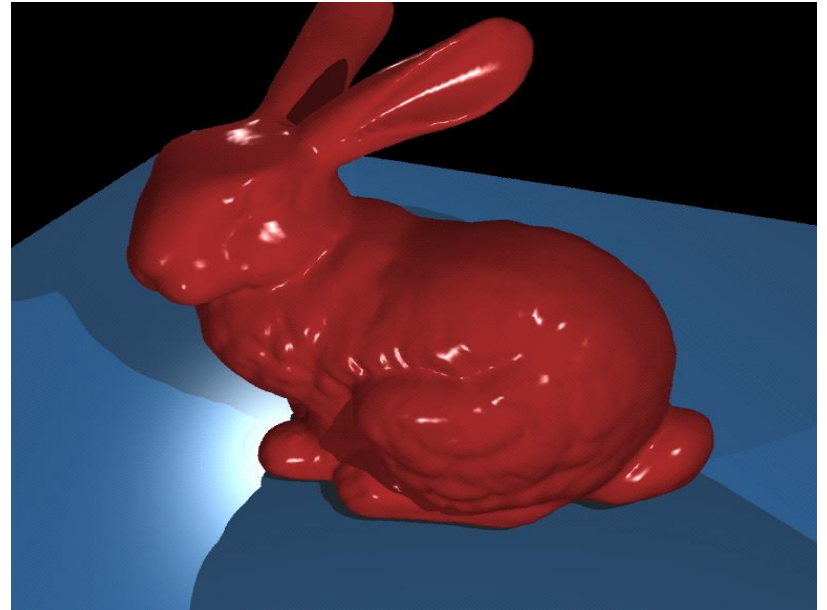
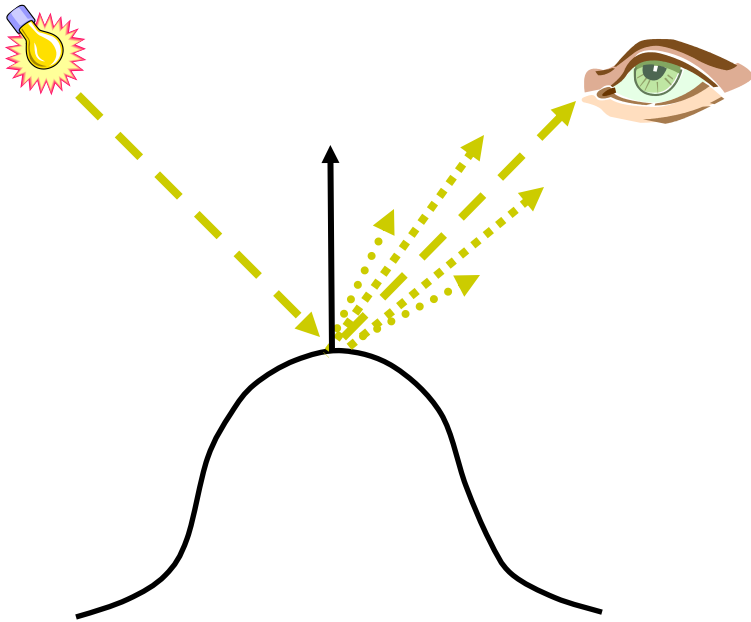
Specular reflectance (mirror)

- When a surface is smooth light reflects in the opposite direction of the surface normal



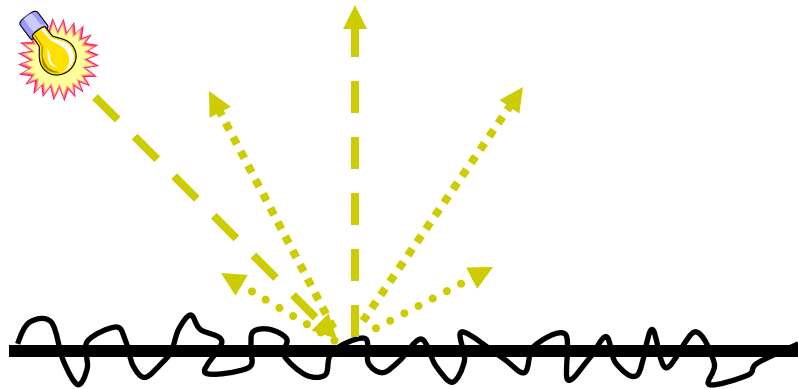
Specular reflectance

- When a surface is slightly rough the reflected light will fall off around the specular direction



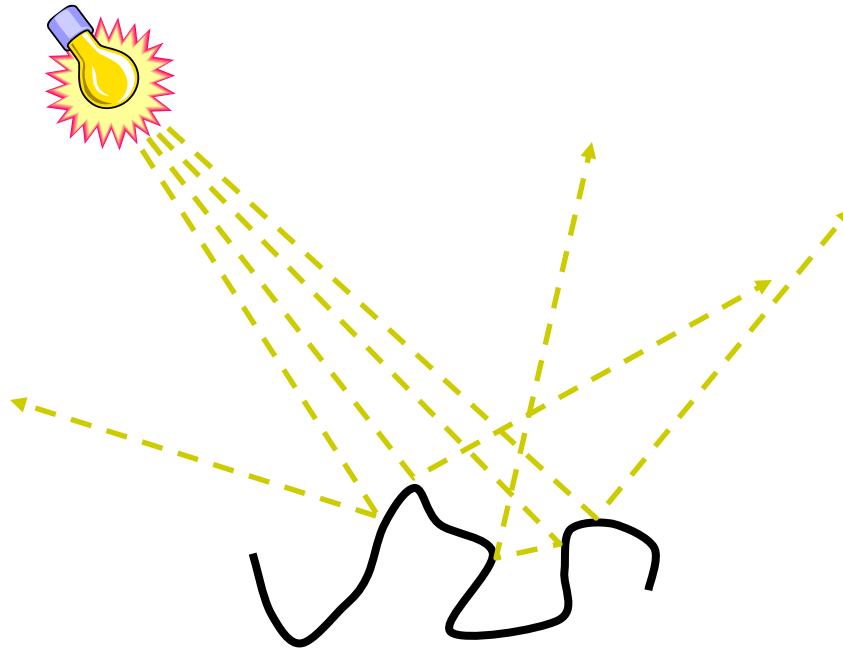
Diffuse reflectance

- When the surface is very rough light may be reflected equally in all directions



Diffuse reflectance

- When the surface is very rough light may be reflected equally in all directions

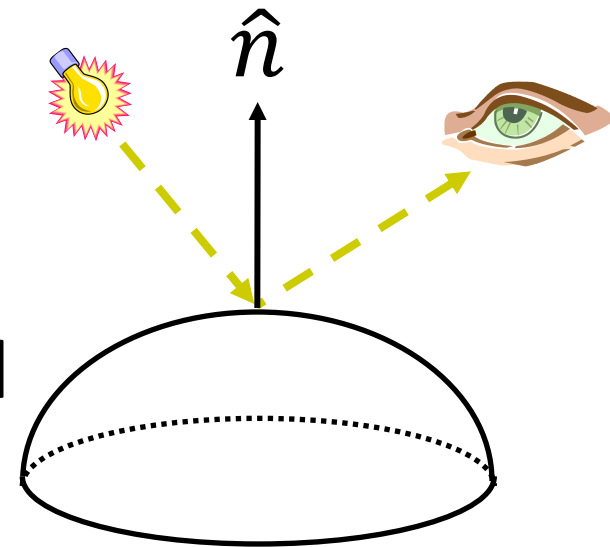


BRDF

- *Bidirectional Reflectance Distribution Function*

$$f(\theta_{in}, \phi_{in}; \theta_{out}, \phi_{out})$$

- Specifies for a unit of incoming light in a direction (θ_{in}, ϕ_{in}) how much light will be reflected in a direction $(\theta_{out}, \phi_{out})$

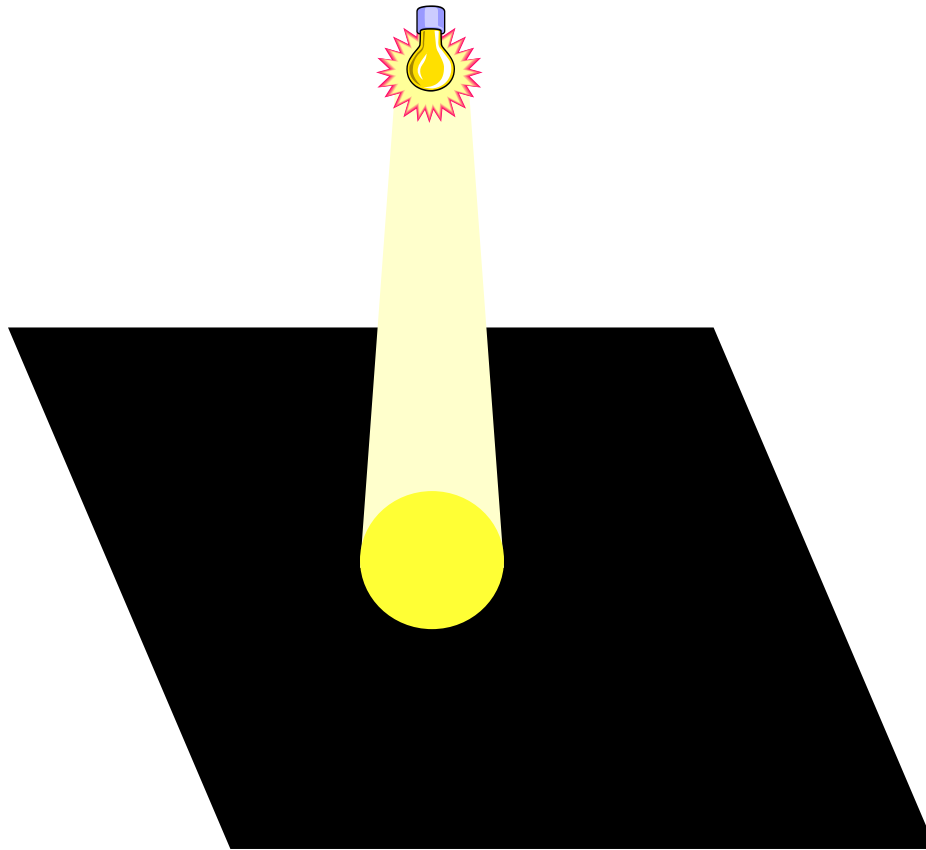


BRDF

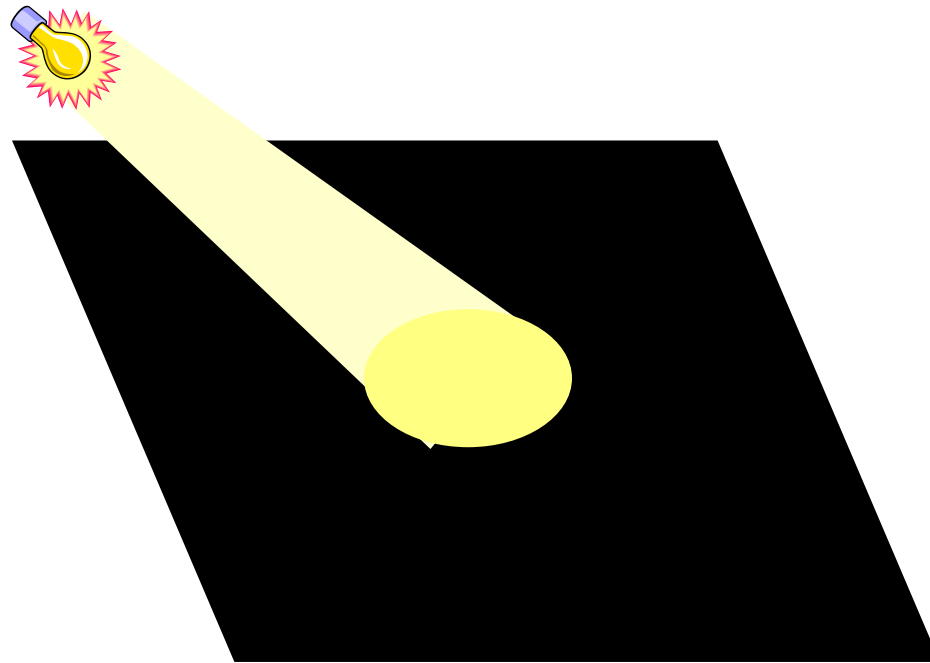


Light from front Light from back

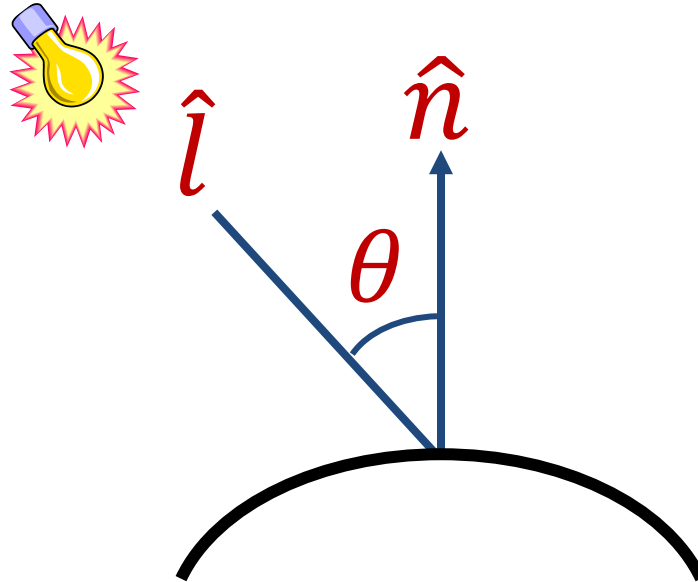
Lambertian reflectance



Lambertian reflectance



Lambert's law



$$I = E\rho \cos \theta \quad (\theta \leq 90^\circ)$$

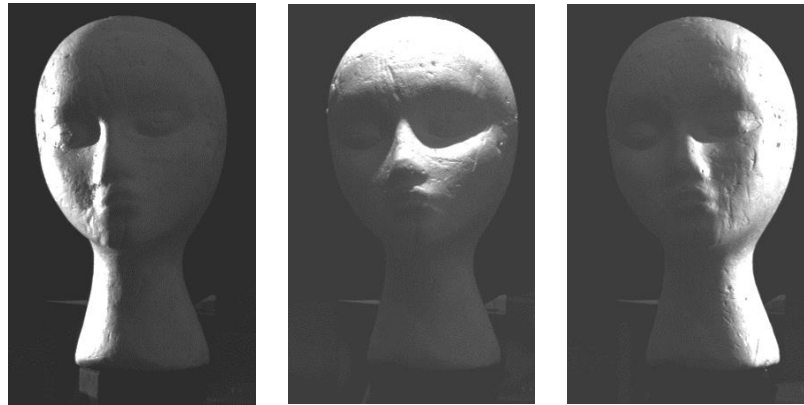
$$I = \vec{l}^T \vec{n} \quad (\vec{l} = E\hat{l}, \vec{n} = \rho\hat{n})$$

Preliminaries

- A surface is denoted $z(x, y)$
- A point on z is $(x, y, z(x, y))$
- The tangent plane is spanned by
 $(1, 0, z_x)$ $(0, 1, z_y)$
- The surface normal is given by

$$\hat{n} = \frac{1}{\sqrt{z_x^2 + z_y^2 + 1}} (-z_x, -z_y, 1)$$

Photometric stereo



- Given several images of the a lambertian object under varying lighting
- Assuming single directional source

$$M = LS$$

Photometric stereo

$$M = LS$$

$$\begin{bmatrix} I_{11} & \cdots & I_{1p} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ I_{f1} & \cdots & I_{fp} \end{bmatrix}_{f \times p} = \begin{bmatrix} l_{1x} & l_{1y} & l_{1z} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ l_{fx} & l_{fy} & l_{fz} \end{bmatrix}_{f \times 3} \begin{bmatrix} n_{x1} & \cdots & \cdots & n_{xp} \\ n_{y1} & \cdots & \cdots & n_{yp} \\ n_{z1} & \cdots & \cdots & n_{zp} \end{bmatrix}_{3 \times p}$$

- We can solve for S if L is known (Woodham)
- This algorithm can be extended to more complex reflection models (if known) through the use of a lookup table

Factorization (Hayakawa)

- Use SVD to find a rank 3 approximation

$$M = U\Sigma V^T$$

- Define $\Sigma_3 = \text{diag}(\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1, \sigma_2, \sigma_3$ are the largest singular values of M

$$\hat{L} = U\sqrt{\Sigma_3}, \quad \hat{S} = \sqrt{\Sigma_3}V^T \quad \text{and} \quad M \approx \hat{L}\hat{S}$$

- Factorization is not unique, since

$$\hat{M} = (\hat{L}A^{-1})(A\hat{S}), \quad A \text{ is } 3 \times 3 \text{ invertible}$$

- We can reduce ambiguity by imposing integrability

Generalized bas-relief ambiguity

(Belhumeur, Kriegman and Yuille)

- Linearly related surfaces: given a surface $z(x, y)$ the surfaces related linearly to z are:

$$\tilde{z}(x, y) = ax + by + cz(x, y)$$

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & c \end{bmatrix} \quad G^{-1} = \frac{1}{c} \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ -a & -b & 1 \end{bmatrix}$$

- Forms a sub-group of $GL(3)$

Bas-relief



Integrability

- Objective: given a normal field $n(x, y) \in S^2$ recover depth $z(x, y) \in \mathbb{R}$

- Recall that

$$n = (n_x, n_y, n_z) = \frac{\rho}{\sqrt{z_x^2 + z_y^2 + 1}} (-z_x, -z_y, 1)$$

- Given n , we set

$$\rho = \|n\| ; p = -\frac{n_x}{n_z}, q = -\frac{n_y}{n_z}$$

Integrability

- Solve

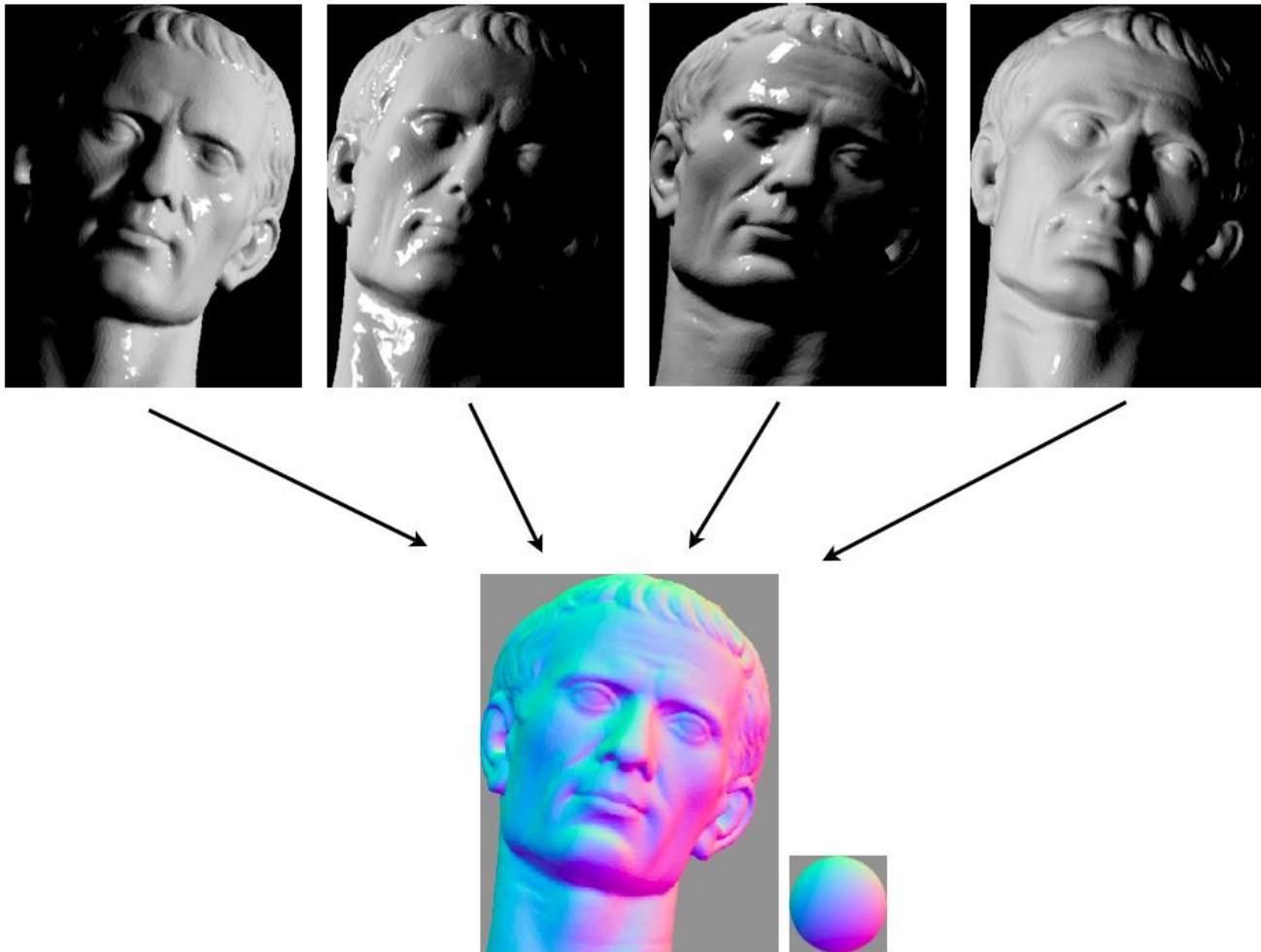
$$\min_{z(x,y)} (z(x+1, y) - z(x, y) - p)^2 + (z(x, y+1) - z(x, y) - q)^2$$

where

$$p = -\frac{n_x}{n_z}, q = -\frac{n_y}{n_z}$$

Photometric stereo with matrix completion

(Wu et al.)

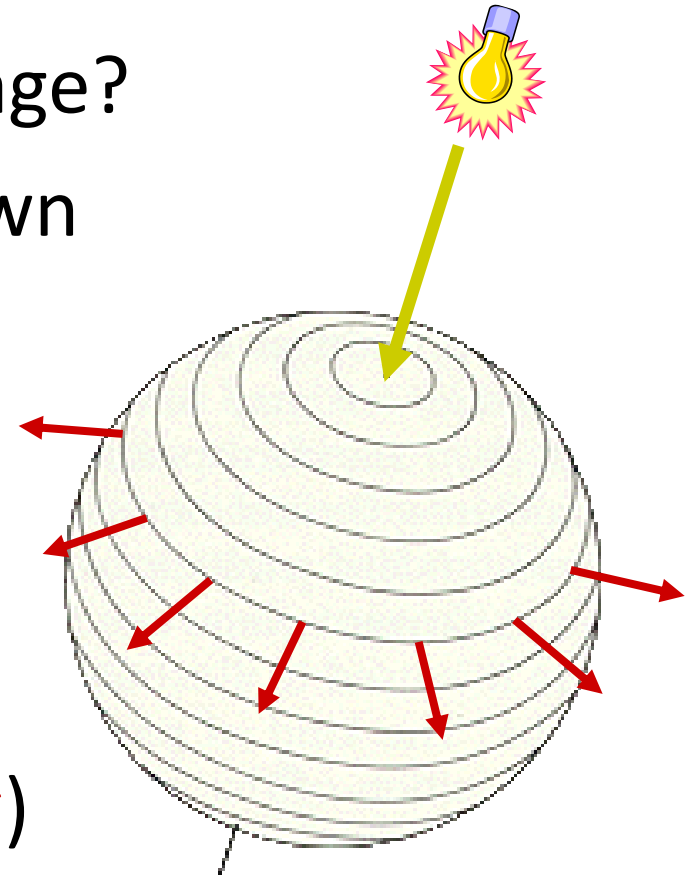


Shape from shading (SFS)

- What if we only have one image?
- Assuming that lighting is known and albedo is uniform

$$I = \vec{l}^T \vec{n} \propto \cos \theta$$

- Every intensity determines a circle of possible normals
- There is only one unknown (z) for each pixel
(since uniform albedo is assumed)



Shape from shading

- Denote $\mathbf{n} = (-z_x, -z_y, 1)$
- Then

$$E = \frac{l^T \mathbf{n}}{\|\mathbf{n}\|}$$

- Therefore

$$E^2 \mathbf{n}^T \mathbf{n} = \mathbf{n}^T l l^T \mathbf{n}$$

- We obtain

$$\mathbf{n}^T (l l^T - E^2 I) \mathbf{n} = 0$$

- This is a first order, non-linear PDE (Horn)

Shape from shading

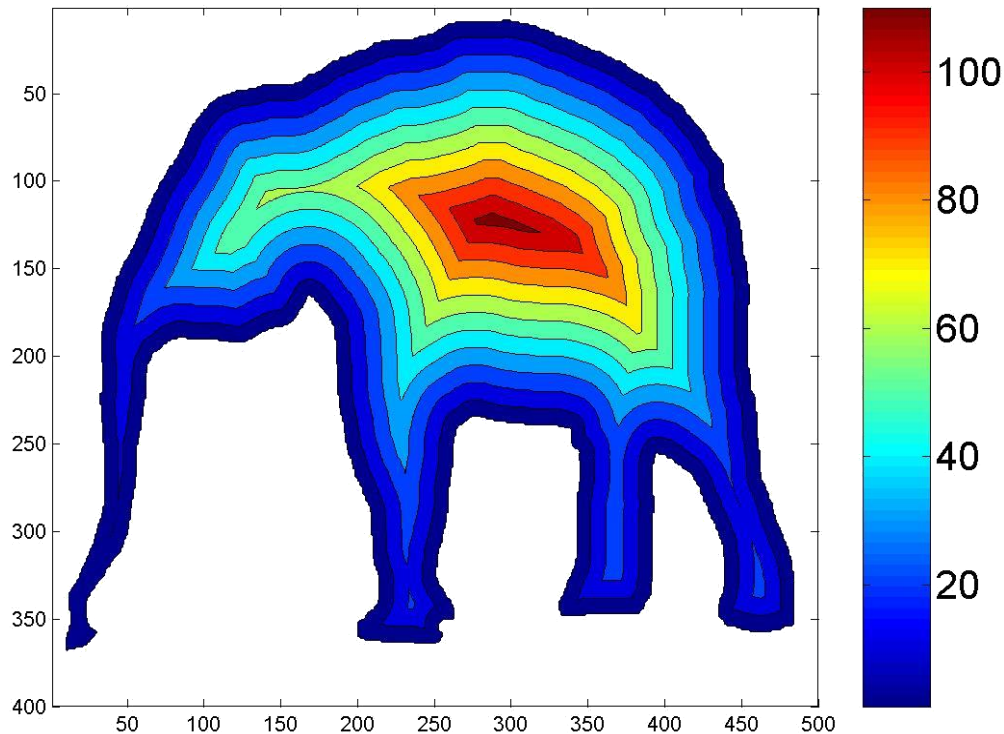
- Suppose $l = (0,0,1)$, then

$$z_x^2 + z_y^2 = \frac{1}{E^2} - 1$$

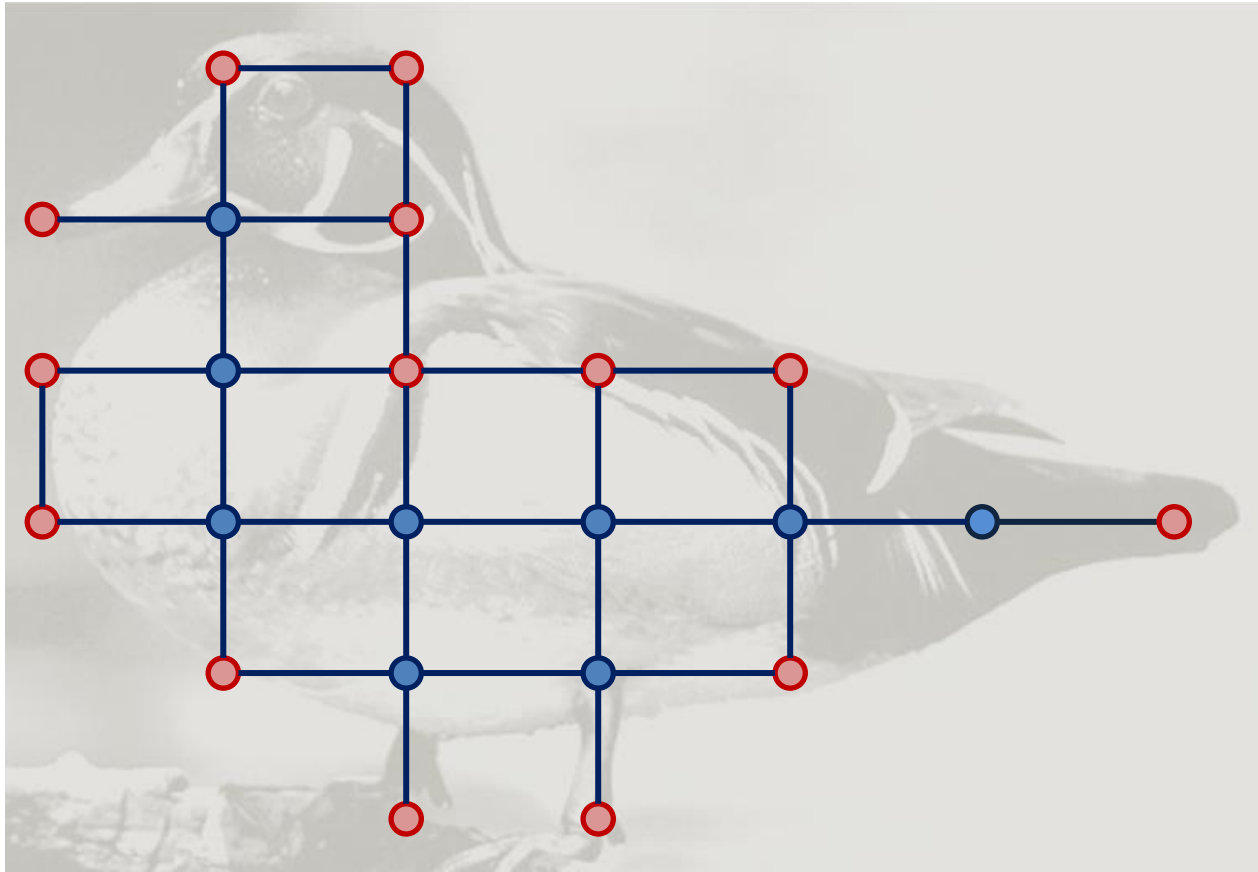
- This is called an *Eikonal* equation

Distance transform

- The distance of each point to the boundary



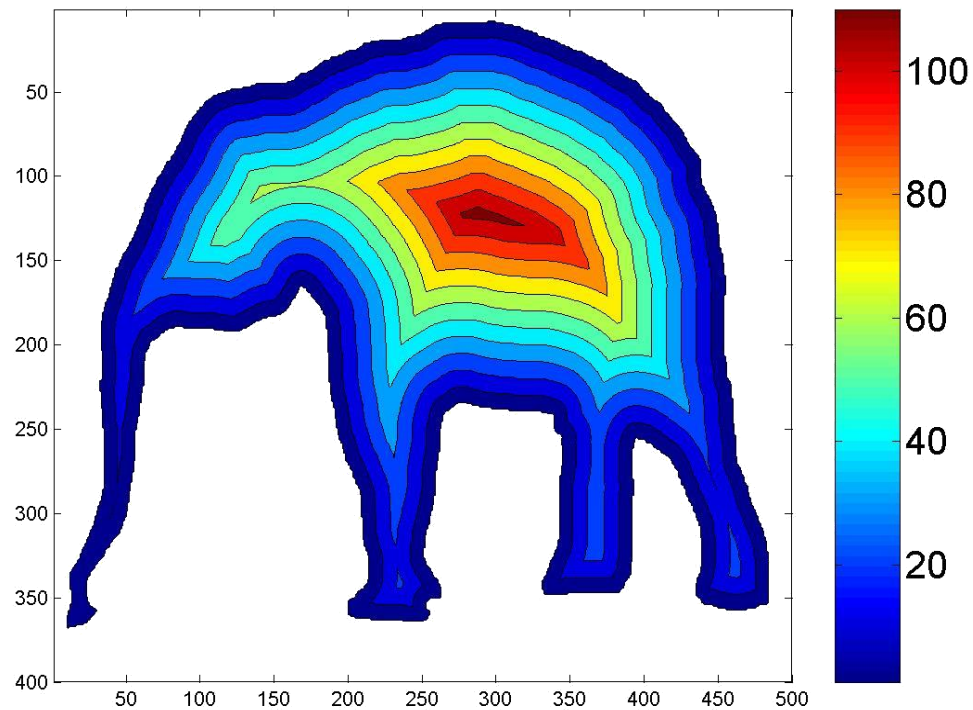
Shortest path



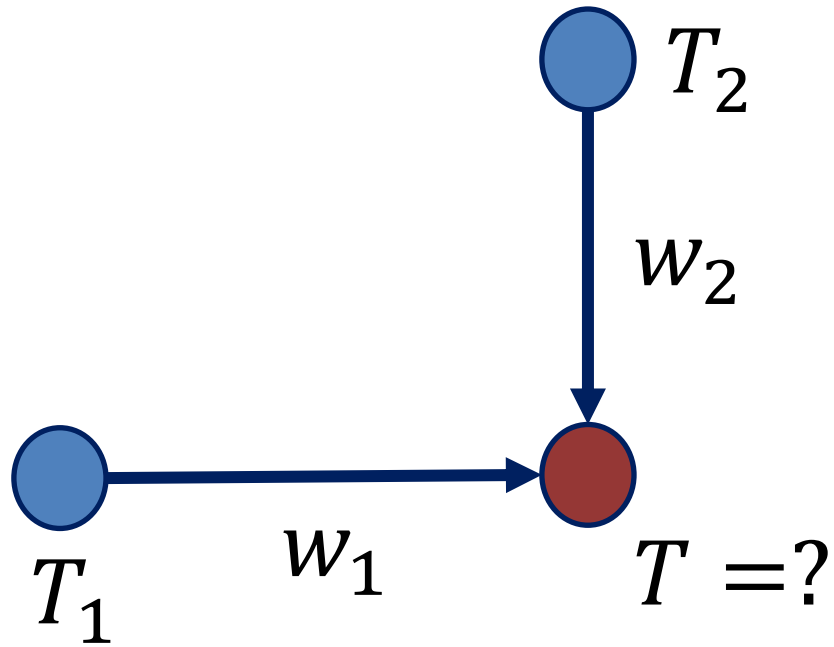
Distance transform

- Posed as an Eikonal equation

$$\|\nabla z\|^2 = z_x^2 + z_y^2 = 1$$



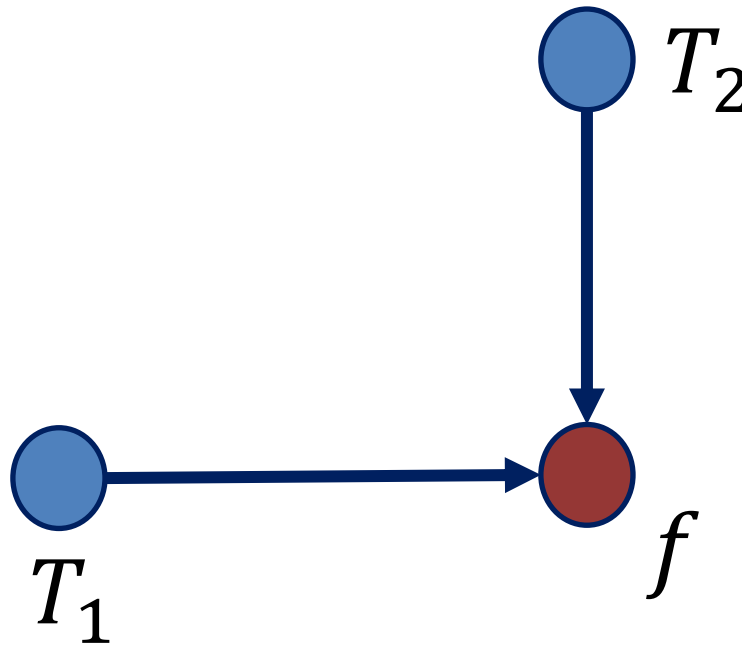
Update for shortest path



$$T = \min\{T_1 + w_1, T_2 + w_2\}$$

Update for fast marching

(Tsitsiklis, Sethian)



$$T = \begin{cases} T_1 + T_2 + \sqrt{2f^2 - (T_1 - T_2)^2} & \text{if real} \\ f + \min\{T_1, T_2\} & \text{otherwise} \end{cases}$$

SFS Solution

(Kimmel & Sethian)

- Lambertian SFS produces an eikonal equation

$$\|\nabla z\|^2 = z_x^2 + z_y^2 = \frac{1}{E^2} - 1$$

- Right hand side determines “speed”
- Boundary conditions are required: depth values at local maxima of intensity and possibly in shape boundaries
- The case $l \neq (0,0,1)$ is handled by change of variables

Example



Summary

- Understanding the effect of lighting on images is challenging, but can lead to better interpretation of images
- We considered Lambertian objects illuminated by single sources
- We surveyed two problems:
 - Photometric stereo
 - Shape from shading

Challenges

- General reflectance properties
 - Lambertian
 - specular
 - general BRDFs
- Generic lighting
 - multiple light sources (“attached shadows”)
 - near light
- Cast shadows
- Inter-reflections
- Dynamic scenes
- Local approaches (eg. direction of gradients)

Can we hope to model this complexity?