600.363/463 Algorithms
Assignment 2
Due Sept 23, 2013

I. By applying the master theorem solve the following recurrences. For the base cases, assume that T is $O(1)$.

1. $T(n) = 25T(n/5) + n^{2.1}$
2. $T(n) = 25T(n/5) + n^{1.5}$
3. $T(n) = 25T(n/5) + n^2$

II. Solve the following recurrence by successive substitutions or by induction. For the base cases, assume that T is $O(1)$.
$T(n) \leq 25T(n/5) + n^2 \log n$.

III. The *element distinctness* problem consists of testing whether a given set of n numbers have no duplicates. Design an $O(n \log n)$ step comparison-based algorithm for this problem.

IV. In the selection problem (finding the k^{th} smallest element), if we group the n elements into $n/3$ groups each of 3 elements and make appropriate changes to the algorithm, derive the speed of the resulting algorithm. Repeat it when each group consists of 7 elements.

V. (BONUS PROBLEM) For the element distinctness problem derive a lower bound of $\Omega(n \log n)$.