600.363/463 Algorithms
Assignment 4
Due October 7, 2013

I. A weight balanced tree is a binary tree such that at every nonleaf node, v, the number of nodes in each of its subtrees is at least one fifth of the total number of nodes in the subtree with root v. Modify the optimal binary search tree algorithm (that minimizes the expected weight) such that the resulting tree is a weight balanced tree. That is, the optimization has to be limited to weight balanced trees.

II. A double-ended queue (deque) is a linear list such that inserts and deletes can be performed at either end. As usual, the space used by the deque is the maximum size of the deque up to that instant. Implement the deque on a RAM such that if the deque makes use of $S(n)$ space and performs $T(n)$ steps, the RAM uses $O(S(n))$ space and performs $O(T(n))$ steps.

III. Consider the following variant of the longest common subsequence problem. Instead of each symbol having a weight of 1, each sequence of 2 symbols has a positive weight. Let this weight function be δ. Then for any sequence $b_1, b_2, \cdots, b_k$ the total weight is $\sum_{i=1}^{k-1} \delta(b_i, b_{i+1}) + \delta(b_k, b_k)$. For example, if $\delta(a,a) = 1, \delta(a,b) = 2, \delta(b,a) = 1, \delta(b,b) = 4$, then the total weight of $abbba$ is $2+4+4+1+1 = 12$.

Design a dynamic programming algorithm for computing a common subsequence with maximum total weight for any 2 given sequences.

IV. (BONUS PROBLEM) Assume that in the binary search tree problem, each $q_i = 0$. (This is meant as a simplifying assumption.) Then the algorithm we discussed computed a binary search tree that minimized $\sum_{i=1}^{n} p_i(d_i + 1)$. Design an algorithm that computes the binary search tree that minimizes $\sum_{i=1}^{n} p_i(d_i + 1)^2$. Estimate its speeed.

V. (BONUS PROBLEM) Implement a queue on a RAM such that the RAM makes use of $O(S(n))$ space and performs $O(1)$ steps per step of the queue.