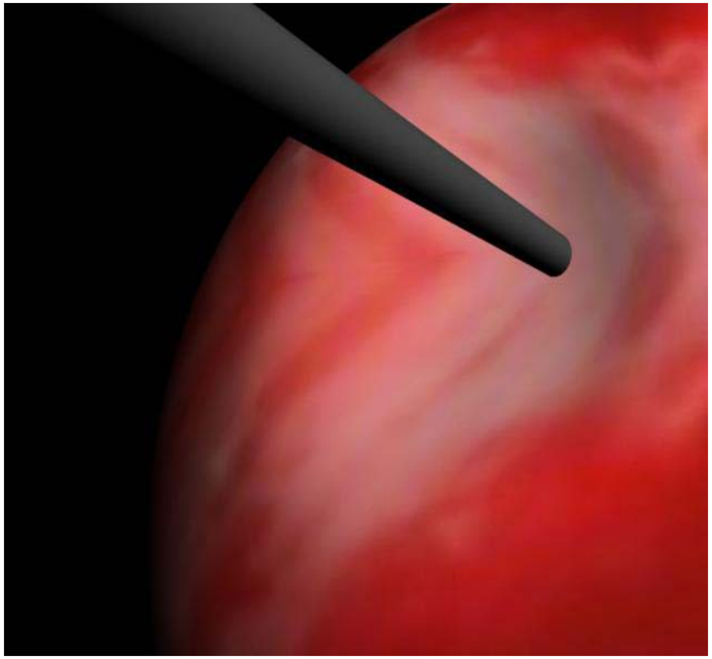
Haptic Systems
530-655

Mohsen Mahvash
Lecture 9
20/1/06

Haptic rendering of deformable bodies

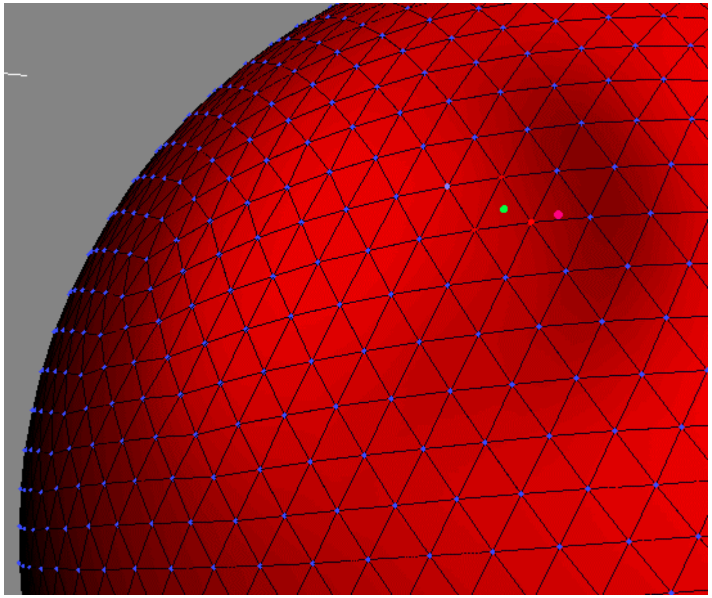
- Problem statement
- Theory of deformation
- Methods for calculating deformation
- Finite Element Method
- Pre-computation
- Tool-based pre-computation
- References

Problem statement



- Real-time computation of deformation (30 Hz for graphic rendering)
- Real-time computation of contact forces (1000 Hz for haptic rendering

Objects with surface meshes

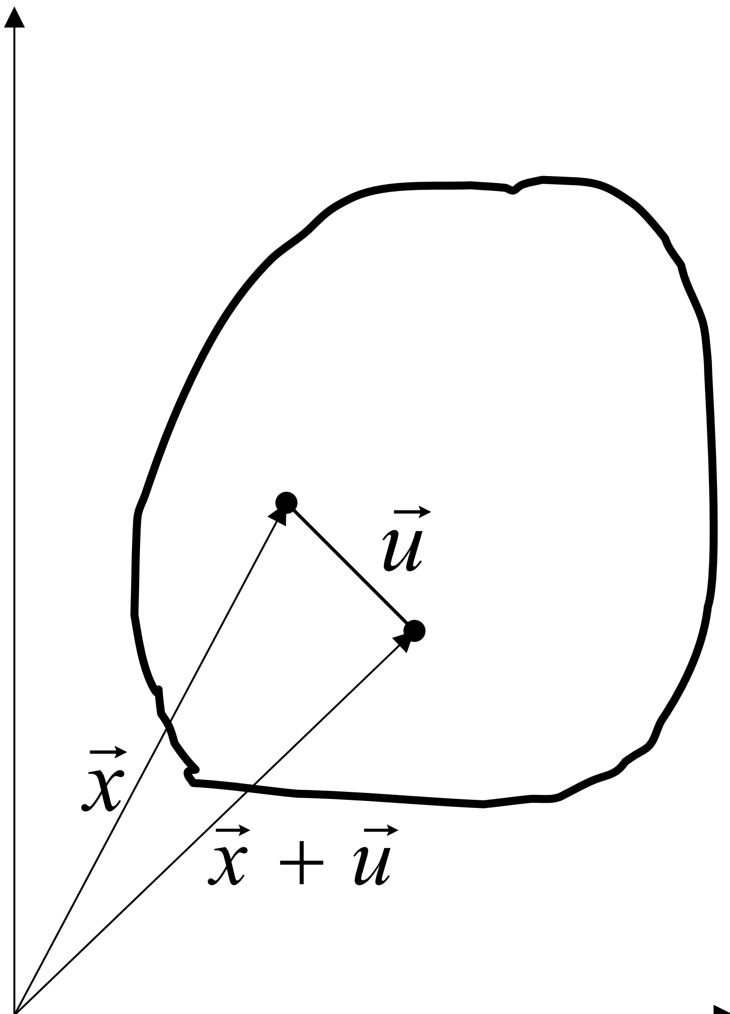


Graphic rendering: updating the positions of the vertices of the object (for example 500 vertices)

Haptic rendering: updating the applied forces to the tip of a haptic device

Theory of Deformation

Displacement: $\vec{u}(\vec{x}) = [u(\vec{x}), v(\vec{x}), w(\vec{x})]$



Strain: $\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \varepsilon_z = \frac{\partial w}{\partial z}$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\varepsilon = Bu$$

Material: $\sigma = D\varepsilon$

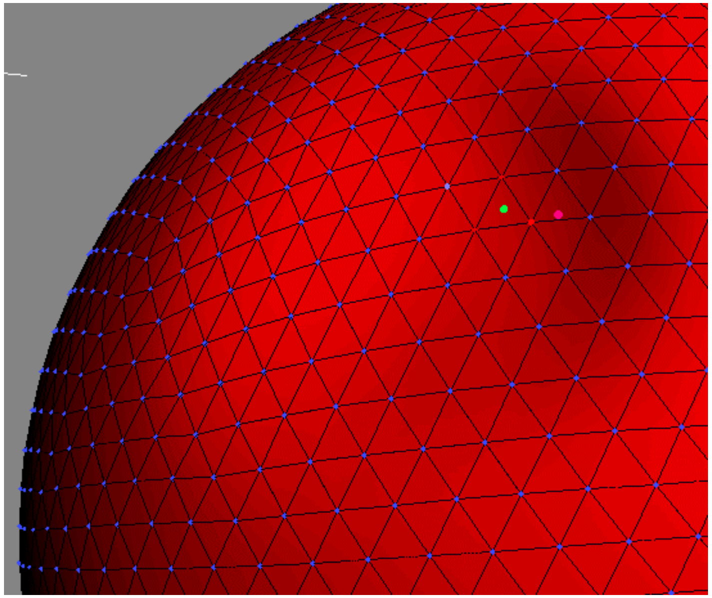
Stress: $T^n = \frac{\Delta F}{\Delta A}$ $\quad T^n = \sigma \quad n$

Energy: $E(\vec{u}) = 1/2 \iiint \varepsilon^T \sigma dV$

See Ref 1,5

Computational Approaches

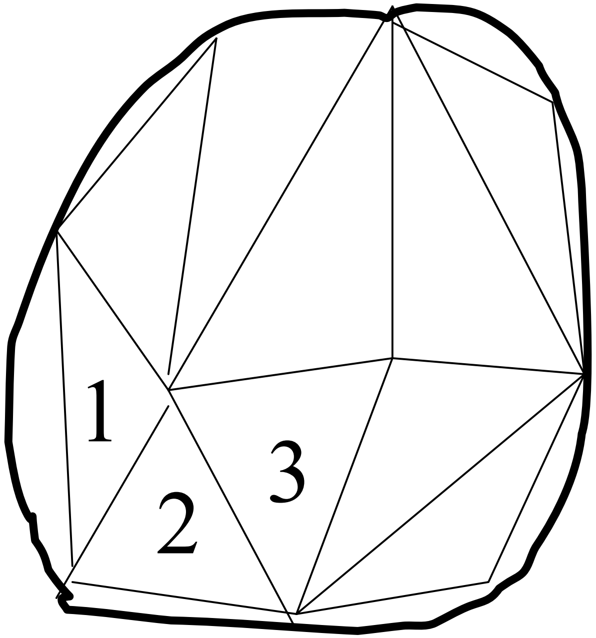
- Continuum models
  - Finite element method
  - Boundary element method
  - Finite difference method

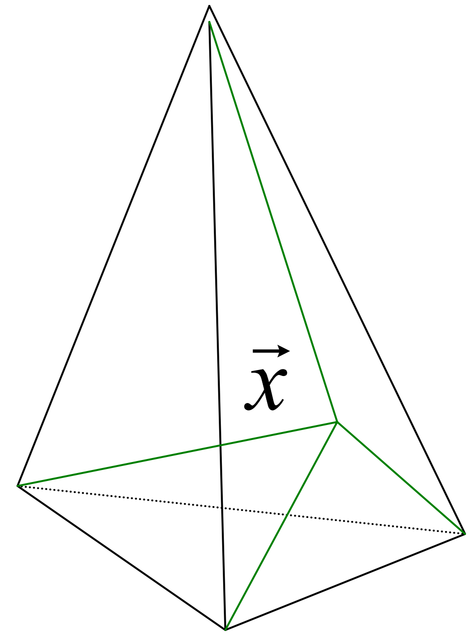


- Mass-spring method

Finite Element Method

Body





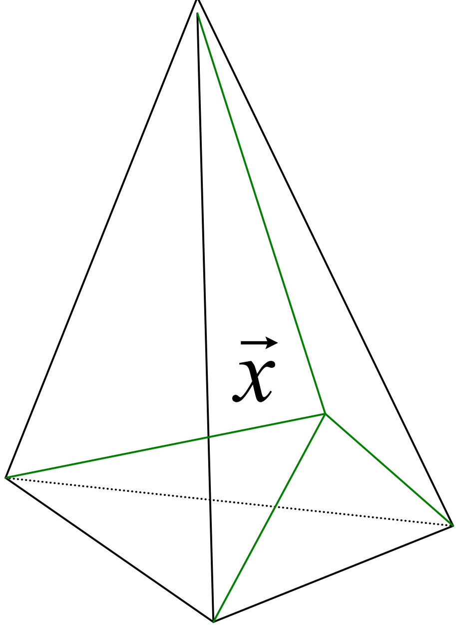
$$\vec{u}(\vec{x}) = \sum_{i=1}^{4} N_i^e(\vec{x})\vec{u}_i^e(\vec{x})$$

$$E(\vec{u}) = 1/2 \iiint u^e B^{e^T} D B^e u^e dV - \iint f^e du^e$$

Virtual work: $\partial E(\vec{u}) = 0 \quad K^e u^e = f^e$

See Ref 1

Surface nodes responses

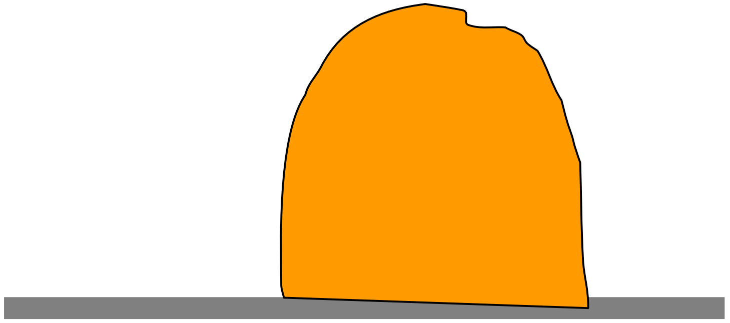


Element responses $K^e u^e = f^e$

Body responses $\underline{K}\,\underline{u} = \underline{f}$

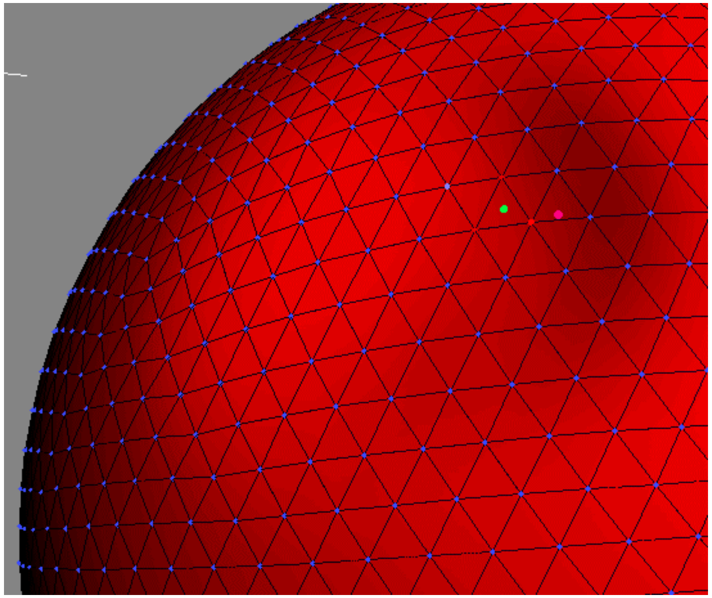
Surface responses $\underline{K}_s \underline{u}_s = \underline{f}_s$

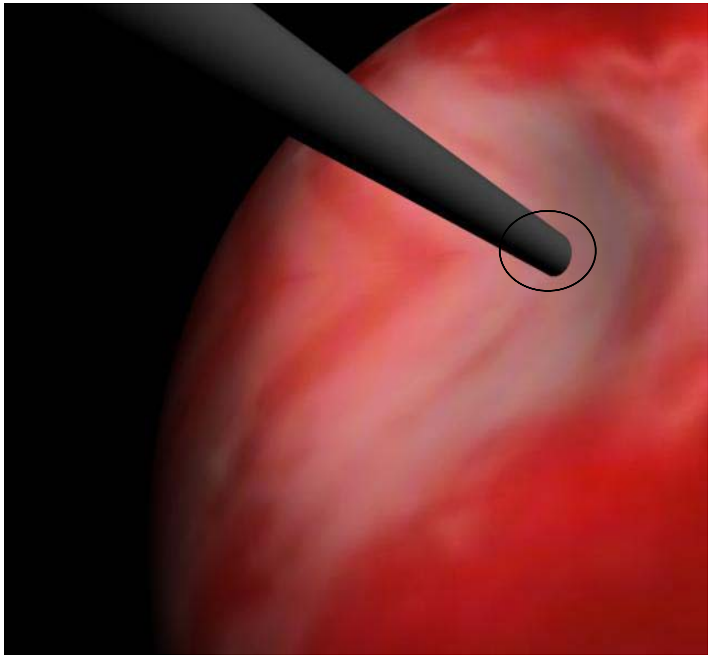
Free node responses $\underline{K}_f \underline{u}_f = \underline{f}_f$



Pre-computation

- Define free nodes and fixed nodes
- Calculate deformation response for unit displacements at each free node
- Derive a reduced-order linear system that directly relates the nodal displacements for local contact region
- Calculate contact forces





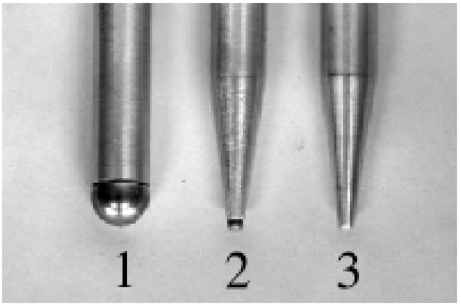
See Ref 2

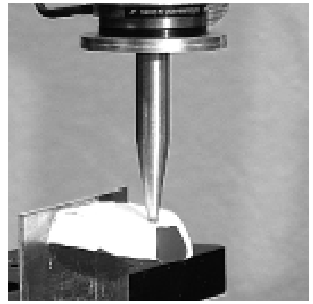
Boundary Element Method

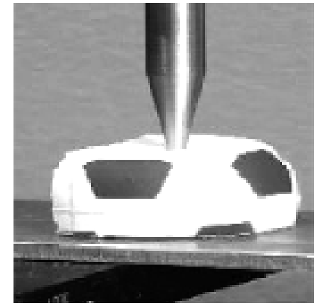
- Define the integral equation of elasticity over boundary
- Divide the boundary into elements
- Use FEM to solve the boundary integral equation
- Derive an algebraic equation among nodal values over boundary
- Pre-compute nodal force responses
- Calculate nodal displacements at contact area and contact forces

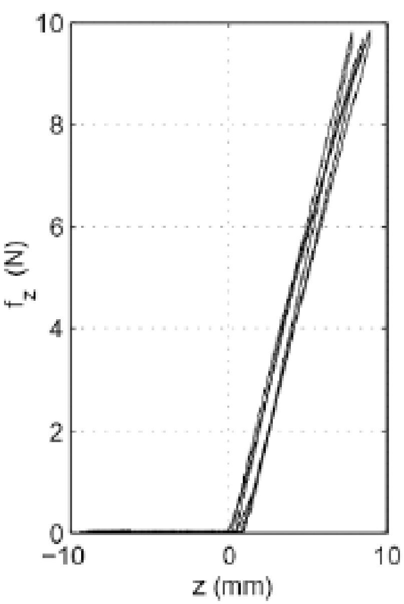
See Ref 3

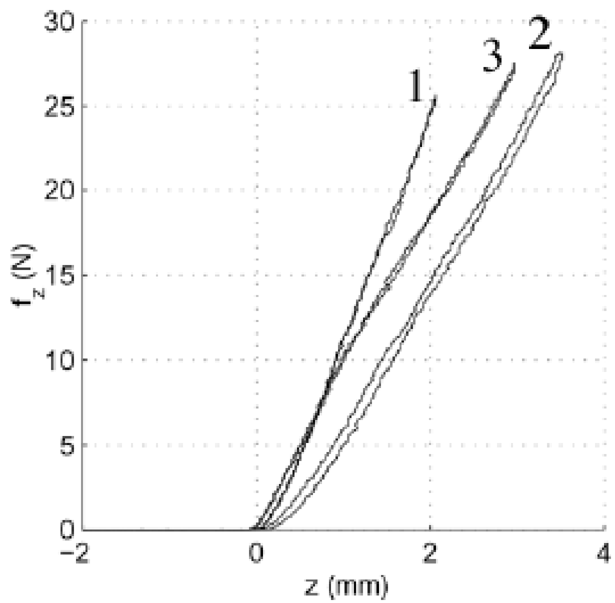
Tool-based pre-computation (Tests)

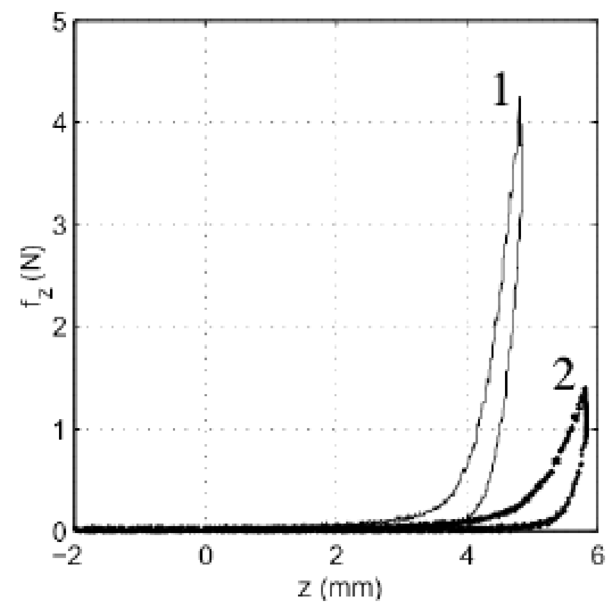




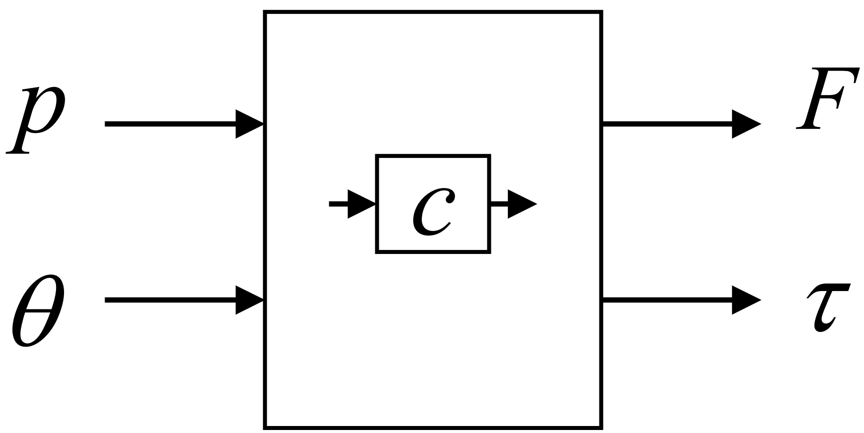




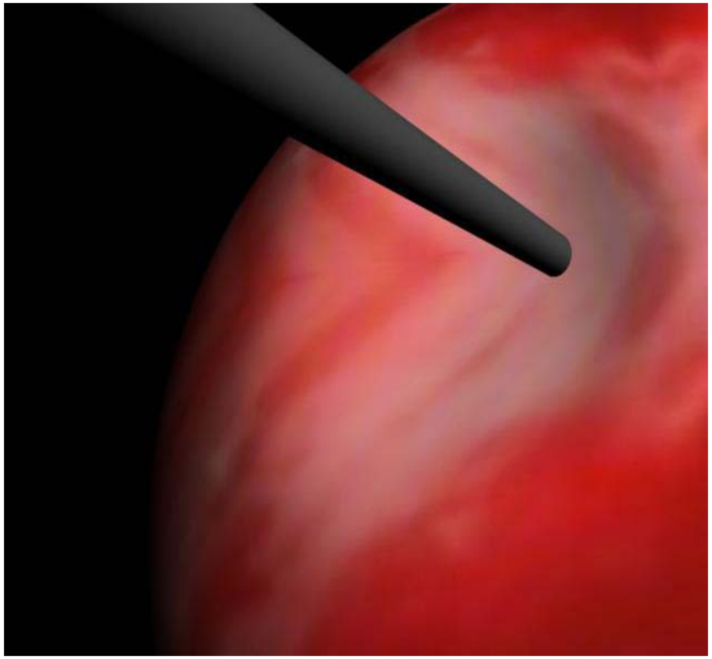




A system view for tool-contact simulation



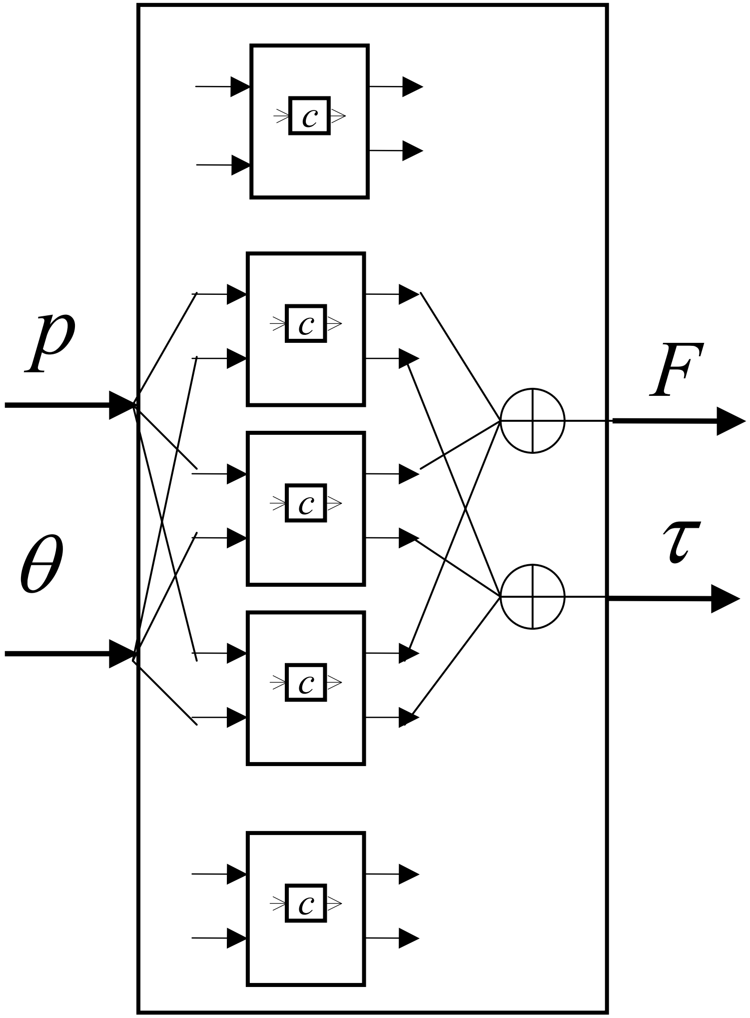
$$F = F(p, c, \theta)$$

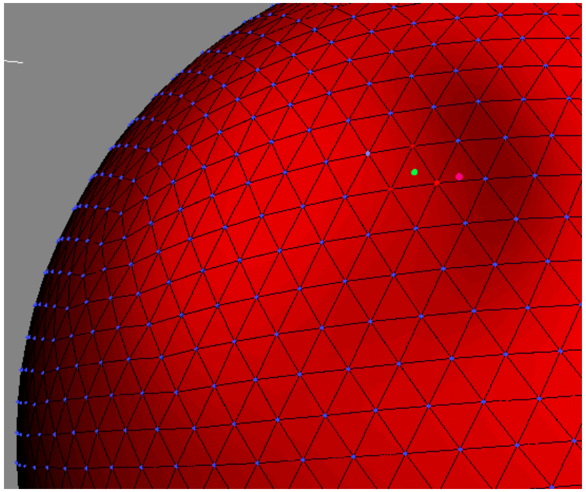


Contact forces = A mathematical function of a few external and internal variables.

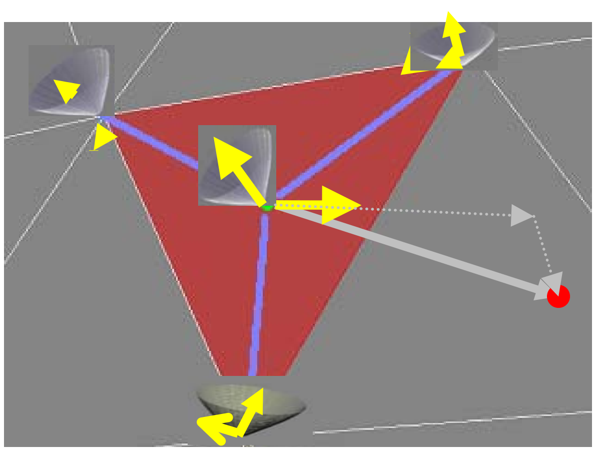
Local models

$$F = F(p, c, \theta)$$





Determination of forces at nodes of the body surface



Online interpolation of forces at nodes of the body surface

References

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Monday

Haptic rendering of deformation and cutting