We have shown in the class that the coloring MC is ergodic. Now we show that its stationary distribution is uniform, i.e., \( \pi = \left( \frac{1}{c}, \frac{1}{c}, \ldots, \frac{1}{c} \right) \) if \( c \) is the number of states (different colorings). We show that the detailed balance condition is satisfied. Note that if 2 colorings differ by more than 1 vertex, there is no transition between them. Let 2 colorings.

\[ C_1 \neq C_2 \text{ differ by 1 vertex, and let the vertex be } \mathbf{u}. \text{ Let } C_1(\mathbf{u}) = \alpha \neq C_2(\mathbf{u}) = \beta. \text{ For a coloring to be valid, no neighbor of } \mathbf{u} \text{ (in graph G) can have the color } \alpha \text{ or } \beta \text{ in } C_1 \text{ (and hence } C_2). \text{ Hence to transition from } C_1 \text{ to } C_2, \text{ we need to pick coordinate of vertex } \mathbf{u} \text{ (prob } \frac{1}{n} \text{ and color } \beta \text{ (prob } \frac{1}{3d})). \text{ Hence the prob is } \frac{1}{n(3d)}. \]

Similarly, \( S \to C_1 \) transition prob is \( \frac{1}{n(3d)} \).

\[ \begin{align*}
\Pi_1 P_{12} &= \frac{1}{n(3d)} \\
\Pi_2 P_{21} &= \frac{1}{n(3d)} \\
\text{Hence } \Pi_1 P_{12} &= \Pi_2 P_{21}.
\end{align*} \]

I don't think it is possible.

The expected dist. doesn't go down.

Skip the prob.

When we included all edges to capture 1 bit complementarily, after one step of evolution, the max dist is no more than 1. However, if we include edges
corresponding to a spanning tree, after one step, the distortion can become significantly more (even though the Hamming dist is ≤ 2) than 1. Then the expected distortion will not decrease.

In fact, I conjecture that this property is true for every spanning tree. It might not be too hard to prove it, but it is not a significant problem to waste a lot of time on it.