Haptic rendering of deformable bodies

- Problem statement
- Theory of deformation
- Methods for calculating deformation
- Finite Element Method
- Pre-computation
- Tool-based pre-computation
- References
Problem statement

- Real-time computation of deformation (30 Hz for graphic rendering)
- Real-time computation of contact forces (1000 Hz for haptic rendering)

Objects with surface meshes

Graphic rendering: updating the positions of the vertices of the object (for example 500 vertices)

Haptic rendering: updating the applied forces to the tip of a haptic device
Theory of Deformation

Displacement: \( \vec{u}(\vec{x}) = [u(\vec{x}), v(\vec{x}), w(\vec{x})] \)

Strain:
\[ \varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \varepsilon_z = \frac{\partial w}{\partial z} \]
\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \]
\[ \varepsilon = B u \]

Material:
\[ \sigma = D \varepsilon \]

Stress:
\[ T^n = \frac{\Delta F}{\Delta A} \]
\[ T^n = \sigma \quad n \]

Energy:
\[ E(\vec{u}) = \frac{1}{2} \iiint \varepsilon^T \sigma dV \]

See Ref 1, 5
Computational Approaches

- Continuum models
  - Finite element method
  - Boundary element method
  - Finite difference method

- Mass-spring method
Finite Element Method

Body

\[ \mathbf{u}(\mathbf{x}) = \sum_{i=1}^{4} N_i^e(\mathbf{x}) \mathbf{u}_i^e(\mathbf{x}) \]

\[ E(\mathbf{u}) = \frac{1}{2} \iiint u^e B^{eT} DB^e u^e \, dV - \iint f^e du^e \]

Virtual work:
\[ \partial E(\mathbf{u}) = 0 \quad K^e u^e = f^e \]

See Ref 1
Surface nodes responses

Element responses \( K^e u^e = f^e \)

Body responses \( K u = f \)

Surface responses \( K_s u_s = f_s \)

Free node responses \( K_f u_f = f_f \)
Pre-computation

- Define free nodes and fixed nodes
- Calculate deformation response for unit displacements at each free node
- Derive a reduced-order linear system that directly relates the nodal displacements for local contact region
- Calculate contact forces

See Ref 2
Boundary Element Method

- Define the integral equation of elasticity over boundary
- Divide the boundary into elements
- Use FEM to solve the boundary integral equation
- Derive an algebraic equation among nodal values over boundary
- Pre-compute nodal force responses
- Calculate nodal displacements at contact area and contact forces

See Ref 3
Tool-based pre-computation (Tests)
A system view for tool-contact simulation

\[ F = F(p, c, \theta) \]

Contact forces = A mathematical function of a few external and internal variables.
Local models

\[ F = F(p, c, \theta) \]

Determination of forces at nodes of the body surface

Online interpolation of forces at nodes of the body surface
References


Monday
Haptic rendering of deformation and cutting