Due Date 9/Dec/2016. Fall 2016. Homework 3

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Due on Friday, Dec 9 2016. Email pdf file to Drew at `reisinger@cogsci.jhu.edu` **and name your file** `<firstname>-<lastname>-hw3.pdf`

Question 1. Theory. Gibbs Distribution. Gibbs Sampling

Briefly describe Markov Random Fields (MRF) and how they can be used to design vision models which include spatial context.

The Ising model is specified by a Gibbs Distribution $P(\vec{S}|\vec{I}) = \frac{1}{Z}\exp\{-E(\vec{S};\vec{I})\}$ where the energy $E(\vec{S};\vec{I})$ can be expressed by:

$$E(\vec{S};\vec{I}) = \sum_{x}(S(x) - I(x))^2 + \lambda \sum_{x}\sum_{y \in Nbh(x)}(S(x) - S(y))^2. \quad (1)$$

Here, $Nbh(x)$ denotes the set of pixel indices neighboring x, $S(x) \in \{0, 1\}$ (also called the *state*), and $I(x) \in [0, 1]$ (the *image*). Describe how this model captures spatial context. What is the likelihood distribution for this model? The prior?

Derive the Gibbs sampling distribution for this model. Why is it possible to sample from the conditional distribution $P(S(x)|\{S(y) : y \in Nbh(x)\})$? Why is it impossible to sample directly from the full joint distribution $P(\vec{S}|\vec{I})$? What theoretical results guarantee that Gibbs sampling will converge to samples from the Gibbs distribution (do not need details or derivations, just broad results).

How does Gibbs sampling relate to probabilitic neural models like integrate and fire models?

Question 2. Mean Field Theory

Describe the mean field theory approximation for the Ising model $P(\vec{I}|\vec{S})$. What is $Q(\vec{S})$ and how is the Kullback-Leiber divergence used as a measure of similarity between $P(.)$ and $Q(.)$? What is the free energy? Derive the update equations for the mean field model. How do they relate to deterministic neural networks? (see notes to lecture 9).

Question 3. Experimental

In this question, you will use Gibbs sampling to apply the Ising model to foreground-background segmentation. See

`http://nbviewer.jupyter.org/github/drew-reisinger/AS.50.375HWFall16/blob/master/Gibbs%20Sampling.ipynb`

Note: The modeling conventions in the notebook are slightly different from those above. In particular, in the code, S takes values ± 1 and not $\{0, 1\}$.