Sunday, October 30, 2016 11:25 AM

Motion lecture 2

In the previous lecture we described the slow-and-smooth model for motion. We now describe how these models can explain the distance fall-off effects which occur in motion capture. We analyze a special case. Note: that this relates to other topics in Machine Learning, such as Kernel Methods, Radial Basis functions, and Gaussian Processes.

We will do the analysis in 1-dimension for simplicity.

$$E[v] = \sum_{i=1}^{N} \left(v(x_i) - u_i\right)^2 + \lambda \int \left\{v(x)\right\}^2 dx + \mu \int \left\{\frac{dv}{dx}(x)\right\}^2 dx + \nu \int \left\{\frac{d^2 v}{dx^2}(x)\right\}^2 dx$$

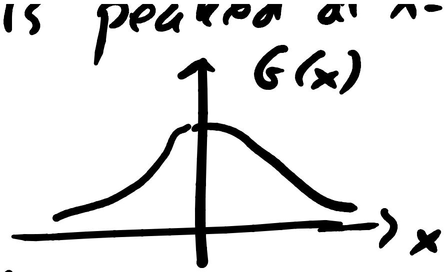
Claim: the solution $\hat{v} = \arg\min E[v]$ can be expressed as:

$$\hat{v}(x) = \sum_{i=1}^{N} \alpha_i G(x - x_i)$$

for a function $G(\,)$ and coefficients $\{\alpha_i\}$.

The function $G(x - x_i)$ is peaked at $x = x_i$

The function $G(x - x_i)$ is peaked at $x = x_i$



and takes the form.

E.g. $G(.)$ can be a Gaussian $G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2}$

where the spatial fall-off depends on the standard deviation σ.

Sub-claim: we can express the slow-and-smoothness terms as

$$\int v(x)\, L\, v(x)\, dx$$

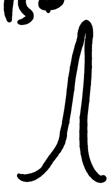
where $L = \lambda - \mu \frac{d^2}{dx^2} + \nu \frac{d^4}{dx^4}$

is a differential operator.

The Green function G is the solution of the equation:

$$L\, G(x) = \delta(x)$$

impulse function at $x = 0$



Green functions are used to solve differential equation $L\, v(x) = \rho(x)$

has a solution $v(x) = \int G(x - x')\rho(x')\, dx'$

How to obtain L ?

$$\mu \int \left(\frac{d v(x)}{dx} \right)^2 dx = \mu \int \frac{d}{dx} \left\{ v(x) \frac{d}{dx} v(x) \right\} dx$$

$$- \mu \int v(x) \frac{d^2}{dx^2} v(x)\, dx$$

The term $\int_a^b \frac{d}{dx} \left\{ v(x) \frac{d}{dx} v(x) \right\} dx$

can be integrated $v(b) \frac{d}{dx} v(b) - v(a) \frac{d}{dx} v(a)$.

Assume that $v(x) = 0$ at the boundaries.

Hence we can rewrite

$$\mu \int \left(\frac{d v(x)}{dx} \right)^2 dx = - \mu \int v(x) \frac{d^2}{dx^2} v(x)\, dx$$

similarly

$$\nu \int \left(\frac{d^2 v(x)}{dx^2} \right)^2 dx = \nu \int v(x) \frac{d^4}{dx^4} v(x)\, dx$$

provided we use boundary conditions

$v(x) \to 0$, $\frac{d v(x)}{dx} \to 0$ on the boundaries.

To determine the α's, substitute

$v(x) = \sum_{i=1}^{N} \alpha_i G(x - x_i)$ into

$$\sum_{i=1}^{N} \left(v(x_i) - u_i \right)^2 + \int v(x)\, L\, v(x)\, dx$$

$$v(x_i) = \sum_{j=1}^{N} \alpha_j G(x_i - x_j)$$

$$\int [illegible] \, dx = \int v(x) \sum^{N} \alpha_i \delta(x - x_i)\, dx$$

$$\int v(x)\, L v(\underline{x})\, d\underline{x} = \int v(\underline{x}) \sum_{i=1}^{N} \alpha_i\, \delta(x - x_i)\, dx$$

$$= \sum_{i=1}^{N} \alpha_i v(x_i) = \sum_{i,j=1}^{N} \alpha_i \alpha_j\, G(x_i - x_j)$$

Hence we solve for $\underline{\alpha}$ by minimizing

$$E[\alpha] = \sum_{i=1}^{N} \left(\sum_{j=1}^{N} \alpha_j\, G(x_i - x_j) - u_i \right)^2 + \sum_{i,j=1}^{N} \alpha_i \alpha_j\, G(x_i - x_j).$$

Summary

Solution $\quad v(x) = \sum_{i=1}^{N} \alpha_i\, G(x - x_i)$

where $G(.)$ is the Green function of the differential operator L. The α's are found by minimizing $E[\alpha]$.

The effect is to smooth the velocity measurements $\{u_1 \ldots u_4\}$, provided they are closer than σ.