Due Date Nov 2. Fall 2017. Homework 3

Prof. Alan Yuille

October 12, 2017

Due on Nov 2 . Submit pdf file on Blackboard by 11:59:59 PM on the due date. Format file name as *firstname*-*lastname*-hw3.pdf. Do not include the iPython notebook code in the pdf submission as it is not required. If you have any questions about the homework, email TA Donald Li: sli97@jhu.edu

Question 1. Decision Theory (28 points)

1. In Bayes Decision Theory, what are priors, likelihood functions, and loss functions? (2points each) What are the formulas for Bayes Risk and the Bayes Decision Rule? (2 points each) What are maximum likelihood (ML) estimation or maximum a posteriori (MAP) estimation? (2 points each) When does the Bayes Decision Rule reduce to them? (2 points each)

2. What are false positives and false negatives? (1 point each) Give a formula for these when the likelihood functions are Gaussians of one variable with the same variance σ^2, but different means μ_T, μ_D. Express the false positives and false negatives in terms of the error function (integrals of Gaussians). (2 points)

3. Why is a first order derivative filter good for edge detection? (2 points) And why is the second order derivative filter less good? (2 points) Given the hierarchical nature of visual processing, and the difficulty of edge detection, what is a good loss function for edge detection? (2 points)

Question 2. Gibbs Sampling (13 points)

The Ising model is specified by a Gibbs Distribution $P(\vec{S}|\vec{I}) = \frac{1}{Z}\exp\{-E(\vec{S};\vec{I})\}$ where the energy $E(\vec{S};\vec{I})$ can be expressed by:

$E(\vec{S};\vec{I}) = \sum_x (S(x) - I(x))^2 + \lambda \sum_x \sum_{y \in Nbh(x)} (S(x) - S(y))^2$.

Here, $Nbh(x)$ denotes the set of pixel indices neighboring x, $S(x) \in \{0, 1\}$ (also called the *state*), and $I(x) \in [0, 1]$ (the *image*).

1. Describe how this model captures spatial context. (2points) What is the likelihood and prior for this model? (2points each)

2. Why is it impossible to sample directly from the full joint distribution $P(\vec{S}|\vec{I})$? (2 points). Compare to sample from the full joint distribution as above, what distribution is sampled from the Gibbs sampling? (2points) Derive the Gibbs sampling distribution for this model. (4 points) What theoretical results guarantee that Gibbs sampling will converge to samples from the Gibbs distribution (do not need details or derivations, just broad results). (3 points)

Question 3. Experimental Section: Edge Detection (14 points)

You will need to use IPython notebook for this project, download the material from either :

`https://github.com/shipui2005/ProbHW3/blob/master/HW3.tar.gz` or

`https://github.com/shipui2005/ProbHW3/blob/master/HW3.zip`

Project: Statistical Edge Detection. Apply Bayes Decision Theory to edge detection.