Probabilistic Models of the Visual Cortex

Fall 2019 Homework 3

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October 28, 2019

Due on Nov 12 before the class. Late homework will not be accepted unless permission is obtained well in advance in documented extenuating circumstances. Please start early on the coding part and make sure the software (i.e. the jupyter notebook) works for you. If you have any questions about the homework, email TA Hongru Zhu: hzhu38@jhu.edu.

Question 1. Gibbs Sampling (13 points)

The Ising model is specified by a Gibbs Distribution $P(\vec{S}|\vec{I}) = \frac{1}{Z}\exp\{-E(\vec{S};\vec{I})\}$ where the energy $E(\vec{S};\vec{I})$ can be expressed by:

$E(\vec{S};\vec{I}) = \sum_x (S(x) - I(x))^2 + \lambda \sum_x \sum_{y \in Nbh(x)} (S(x) - S(y))^2$.

Here, $Nbh(x)$ denotes the set of pixel indices neighboring x, $S(x) \in \{0, 1\}$ (also called the *state*), and $I(x) \in [0, 1]$ (the *image*).

1. Describe how this model captures spatial context. (2points) What is the likelihood and prior for this model? (2points each)

2. Why is it impossible to sample directly from the full joint distribution $P(\vec{S}|\vec{I})$? (2 points). Compare to sample from the full joint distribution as above, what distribution is sampled from the Gibbs sampling? (2points) Derive the Gibbs sampling distribution for this model. (4 points) What theoretical results guarantee that Gibbs sampling will converge to samples from the Gibbs distribution (do not need details or derivations, just broad results). (3 points)

Question 2. Mean Field Theory (15 points)

1. Describe the mean field theory approximation for the Ising model $P(\vec{I}|\vec{S})$. (6 points) What is $Q(\vec{S})$ and how is the Kullback-Leiber divergence used as a measure of similarity between $P(.)$ and $Q(.)$? (6 points)

2. In mean field theory, what is the free energy? (3 points)

Question 3. Boltzmann Machine (12 points)

1. What is the difference between a Boltzmann Machine and a Restricted Boltzmann Machine? (3 points) What is the difference between the hidden and the output variables $(\vec{S}_o, \vec{S}_h)$. $P(\vec{S}) = \frac{1}{Z}\exp\{-E(\vec{S})\}$, where $E(\vec{S}) = C\sum_{ij}\omega_{ij}S_iS_j$. (4 points) What is the update rule for learning the weights $\{\omega_{ij}\}$ in terms of the expected statistics of S_iS_j with respect to the clamped and unclamped distributions. (3 points)

2. Why is it easier to learn these weights for the Restricted Boltzmann Machine. (2 points)

Question 4. Motion and Kalman Filtering (17 points)

1. List one difference between short-range and long-range motion. (1 point) What is one problem of estimating short-range motion? (2 points)

2. To estimate short-range motion, what are the priors terms and their equations? (4 points) Given the prior, if there is no observation, what are the estimated two dimensional velocities? (2 points) What are the estimated velocities if you have an observation term? (2 points)

3. What are the two stages of Bayes-Kalman filter and their equations? (4 points) What is the purpose of the second step? (1 point) Derive the second step equation by using Bayes rule. (1 point)

Question 5. Experimental Section: foreground-background segmentation (16 points)

In this question, you will use Gibbs sampling to apply the Ising model to foreground-background segmentation, IPython notebook is used for this project, follow:
`http://nbviewer.jupyter.org/github/ccvl/VisualCortexCourse/blob/master/HW4/Gibbs%20Sampling.ipynb`

Note: The modeling conventions in the notebook are slightly different from those above. In particular, in the code, S takes values ± 1 and not $\{0, 1\}$.