Divisive normalization

- An important example is the use of probabilistic models (Wainwright & Simoncelli, 2000) to account for divisive normalization. This is a mechanism whereby cells mutually inhibit one another, effectively normalizing their responses with respect to stimulus inputs. Originally developed to explain nonlinear responses to contrast in V1 (Heeger, 1992), divisive normalization has been proposed as a basic cortical computation that underlies various effects of context, as well as higher-level processe,s such as attention (Carandini & Heeger, 2011) .
- The probabilistic approach gives a theoretical justification for divisive normalization in V1. The main idea is that filters with similar preferences for orientation representing nearby spatial locations in a scene have striking statistical dependencies, which can be removed by divisive normalization. Specifically, if we plot the statistics of two linear filters f_c, f_s (center and surround), then the magnitudes of f_c, f_s are coordinated in a straightforward way, which has a characteristic shape of a bow tie.

Modeling divisive normalization using hidden variables

This can be modeled by assuming there are hidden variables ν that affect both responses and hence induces correlation between the responses. For example, ν could represent the local average image intensity, which could affect the response of both filters, but after the filter response, it could be made independent by conditioning on the average intensity. Suppose ν has a prior distribution $P(\nu) = \nu \exp\{-\nu^2/2\}$ for $\nu \geq 0$. We have a pair of filters $\{l_i : i = 1, 2\}$ that are related to Gaussian models $\{g_1 : i = 1, 2\}$. Then we can model the activation of the set of filter responses:

$$P(l_1, l_2) = \int d\nu P(\nu) \prod_{i=1}^{2} P(l_i|\nu, g_i)) P(g_i), \tag{20}$$

where $P(l_i|\nu, g_i) = \delta(l_i - \nu g_i)$. In this model the filter responses are generated by independent processes, g_1, g_2, but then are multiplied by the common factor ν. This is illustrated in the next figure.

Figure for divisive normalization model

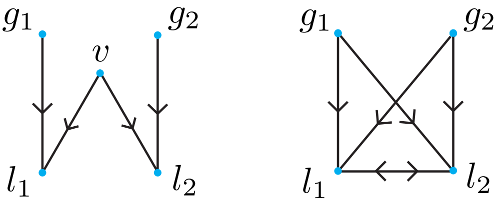

Figure 27: Left: The graphical structure of the divisive normalization model. The filter responses l_1, l_2 are generated from stimuli g_1, g_2 and by the common factor ν. The distributions of l_1, l_2 are factorized if we condition on ν. Right: But if we integrate out ν, then almost all the variables become dependent, as reflected by the complexity of the graph structure.

Divisive normalization model

- In particular, for each filter we can compute $P(g_i|l_1, l_2)$. After some algebra, this is computed to be:

$$P(g_1|l_1, l_2) = \frac{g_1^{-1} \exp\{-\frac{g_1^2 l^2}{2\sigma^2 l_1^2} - \frac{l_1^2}{2g_1^2}\}}{B(0, l/\sigma)}, \quad (21)$$

where $l = \sqrt{l_1^2 + l_2^2}$, and $B(.,.)$ is a Bessel function. To get intuition, note that $g_1 = l_1/\nu$ and $g_1 = l_1/\nu$. So if ν is small, then $|l_1|$ and $|l_2|$ are likely to be small together, while if ν is large, then $|l_1|$ and $|l_2|$ are both likely to be large.

- Assume that the goal of a model unit is to estimate the g_i from the observed filter responses $\{l_i : i = 1, 2\}$, which gives the nonlinear response of the cell. It follows, from analysis above, that

$$E(g_1|l_1, l_2) \propto \mathrm{sign}\{l_1\}\sqrt{|l_1|}\sqrt{\frac{|l_1|}{\sqrt{l_1^2 + l_2^2 + k}}}. \quad (22)$$

The $\sqrt{l_1^2 + l_2^2 + k}$ term sets the gain and performs the divisive normalization.

Application to the tilt illusion

- The model has also been applied to explain the classic tilt illusion in perception (Schwartz et al., 2009; Qiu et al., 2013). In the "simultaneous" tilt illusion, a set of vertically oriented lines appears to tilt right when surrounded by an annulus of lines tilted left–an effect called "repulsion." But for large differences between the center orientation and the surround (tilted left), the center vertical lines can appear to tilt left–an effect called "attraction." In the model, the population of neurons responding to the surround tilted lines contributes to divisive normalizing of the neurons responding to the center stimulus. This results in a change of their neural tuning curves, which, together with the degree of coupling between center and surrounds, accounts for repulsion and attraction.
- The suppressive effect of surround contrast on a central region is an example of local spatial context.

The tilt illusion

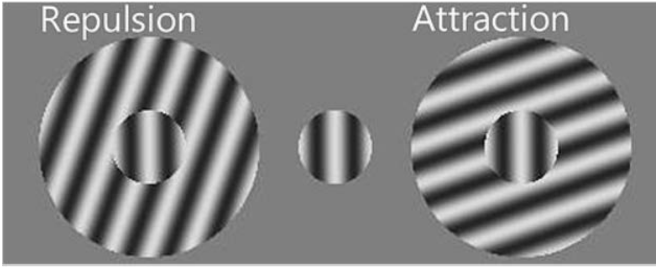

Figure 28: The perceived orientation of a grating pattern can appear to be tilted away from its true orientation due to the presence of surrounding gratings with different orientations. The central circular grating (Center Panel) appear to be tilted to the left (Left Panel) because it is *repulsed* from the orientation of the larger background grating (because the relative orientation is greater than 0 but less than 50 degrees). Conversely it is tilted slightly to the right (Right panel) when it is *attracted* to the background grating (where relative orientation is between 50 and 90 degrees).